Gauge and Ghosts

Guy Hetzroni

This article suggests a fresh look at gauge symmetries, with the aim of drawing a clear line between the *a priori* theoretical considerations involved, and some methodological and empirical non-deductive aspects that are often overlooked. The gauge argument is primarily based on a general symmetry principle expressing the idea that a change of mathematical representation should not change the form of the dynamical law. In addition, the ampliative part of the argument is based on the introduction of new degrees of freedom into the theory according to a methodological principle that is formulated here in terms of correspondence between passive and active transformations. To demonstrate how the two kinds of considerations work together in a concrete context, I begin by considering spatial symmetries in mechanics. I suggest understanding Mach's principle as a similar combination of theoretical, methodological and empirical considerations, and demonstrate the claim with a simple toy model. I then examine gauge symmetries as a manifestation of the two principles in a quantum context. I further show that in all of these cases the relational nature of physically significant quantities can explain the relevance of the symmetry principle and the way the methodology is applied. In the quantum context, the relevant relational variables are quantum phases.

ORCIDs: Hetzroni, https://orcid.org/0000-0002-4402-8713.

1. Introduction

The effectiveness of symmetry considerations in contemporary physics remains puzzling even after a century filled with remarkable achievements. The heart of the matter, it seems to many, is the far-reaching role played by theoretical and mathematical considerations in justifying the way laws are formulated. This exceptional emphasis on *a priori* considerations appears to leave little room, if any, for understanding the form of the laws based on properties of the physical world.

The 'ghosts' in the title of this article refer to the metaphor used by Eugene Wigner to explain the concept of gauge. Wigner ([1964]) compared gauge fields to ghosts that are artificially placed in a physical theory. A change in the coordinates of the ghost does not change the physical situation. Thus the introduction of the ghost creates a formalism in which every physical situation has many equivalent descriptions differing only in the location of the ghost. Gauge symmetries are analogous to the indifference of the theory to the location of the ghost.

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Nevertheless, the concept of gauge has gradually become a cornerstone of modern physics, as a symmetry requirement that determines the form of the fundamental laws of interaction and introduces force-carrying particles. The common Wignerian conception of gauge as an artificial matter of mere representation or convention appears to conflict with its major consequences (Teller [1997]; Martin [2003]). The confusion caused by this conflict is often reflected in the way gauge is presented in textbooks, where, for example, the existence of force-carrying particles and their properties is deduced from the freedom 'to choose one phase convention in Paris and another in Batavia'(Quigg [2013]).

This conflict is the basis of several related foundational questions regarding gauge symmetries that have been raised by different thinkers (Brown [1999]; Lyre [2000]; Teller [2000]; Redhead [2003]; Ben-Menahem [2012]). Most of these worries involve the 'gauge principle' (also referred to as the gauge argument), namely, the introduction of an interaction into a theory of a free field by imposing the demand that a global symmetry would hold as a local symmetry.

This article presents the gauge principle as the result of three elements. The first is a general symmetry principle. The second is an ampliative step where additional degrees of freedom are introduced into an existing theory that does not satisfy the symmetry principle, so as to create a new theory that does. There is nothing *a priori* in this step; it requires empirical input. The third element that turns out to be essential for understanding the gauge principle is the structure of quantum theory.

The symmetry considerations used in gauge theories are presented as a manifestation of a general symmetry principle that is also found in other contexts in physics. Section 2 takes advantage of the clarity and simplicity of spatial degrees of freedom in classical mechanics to formulate this general symmetry principle and demonstrate how it can give rise to arguments about the construction of theories.

It turns out that this symmetry principle is only satisfied in theories that take all relevant degrees of freedom into account. When a successful theory does not satisfy this principle, I suggest seeing this as a sign that its scope has to be extended. Section 2.2 formulates a methodological principle that guides the construction of an extended theory, and presents Mach's principle as a manifestation of this methodology. Section 3 uses a concrete toy model to demonstrate the relation between the symmetry principle and the methodological principle. Section 4 presents the gauge argument as a manifestation of the two principles in a quantum context.

Furthermore, it is demonstrated that in the cases studied the relational nature of fundamental dynamical quantities could be the fact in the world that makes the symmetry principle relevant and the argument successful. This account of gauge symmetries therefore supports the relational approach to gauge recently presented by Rovelli ([2014]).

Section 5 concludes by connecting the presented account to the philosophical discussion regarding the gauge principle, and also discusses the implications of the account given here for the issues of the interpretation of gauge symmetries and the possibility of observing them directly.

2. Spatial Symmetries and Their Methodological Role

Symmetry considerations based on the invariance properties of the dynamics are a powerful tool for constructing theories, as well as for interpreting them. The goal of this section is to motivate and formulate a symmetry principle and a methodological principle. In both cases, the starting point is arguments related to the debate about the ontology of space. Substantivalists consider coordinate systems as a representation of physical space, an actual physical object. Relationists, in contrast, consider them as no more than an auxiliary mathematical structure that allows for a convenient description of the spatial relations between material objects, which they consider as the actual physical quantities. Sections 2.1–2.2 present several key arguments (of both sides) in terms of transformations and symmetries, and generalize to formulate the two principles. Section 2.3 further elaborates on the role of frames of reference.

2.1. A symmetry principle

There are infinitely many ways to label points of space by coordinates. The different coordinate systems are connected by coordinate transformations. Some coordinate transformations may be regarded as nothing more than a change of description, a replacement of one mathematical representation of the set of possible physical situations with a different mathematical representation of the same set.

Which transformations should be regarded as a change of representation is a matter of one's ontological commitments. For example, consider a form of Leibnizian relationism according to which the only meaningful physical variables are relative distances between physical objects. The spatial symmetry group that is entailed by this view is the set of transformations that do not change the values of these relations, namely arbitrary rotations and translations

$$\vec{r} \rightarrow A(t)\vec{r} + \vec{R}(t). \tag{1}$$

(Here $A(t)$ and $\vec{R}(t)$ are a time-dependent orthogonal matrix and a vector in space respectively.)[1] Coordinate transformations that preserve the postulated spatial structure, and are therefore regarded as a change of representation, would be referred to

[1] A different form of relationism, which considers the physical variables to be relative distances together with relative velocities, entails a smaller symmetry group; in this case, rotations have to be excluded and $A(t)$ above becomes fixed in time.

as spatial symmetries. Similarly, coordinate transformations that preserve spacetime structure would be referred to as spacetime symmetries.[2]

Earman ([1989]) suggested the following symmetry principle: any spacetime symmetry of a theory is a dynamical symmetry of the theory. General covariance in general relativity is an ultimate manifestation of the principle. Indeed, the general principle of relativity was presented by Einstein (for example, Einstein [1919]) along the same lines. Einstein's justifications, as well as Earman's arguments for the principle, are fundamentally *a priori* in nature.

This symmetry principle can be similarly employed to argue about Newtonian mechanics. The actual spatial symmetry group of Newton's laws of motion is the Galilean group:

$$\vec{r} \rightarrow A_0\vec{r} + \vec{R}_0 + \vec{v}t. \tag{2}$$

It allows for fixed rotations and translations (characterized by a fixed orthogonal matrix A_0 and a fixed vector R_0), and boosts at some constant velocity $\vec{v}$. This group is significantly smaller than the spatial relationist group represented by Equation (1). Adopting a relationist view of space thus leads to a violation of the symmetry principle. A transition to a rotating reference frame, for example, while preserving all relative displacements, changes the form of the laws of motion through the introduction of centrifugal forces. Therefore the transition cannot be regarded as a change of representation, in contrast to the relationist stance.

This argument against relationism can easily be seen as a version of Newton's bucket argument, one that is expressed in terms of passive rotations. The original argument was used by Newton ([1999]) to support substantivalism. Once the postulated ontology includes absolute space, a transformation of the coordinates of the objects (such as the above rotation) does not stand for a change of representation, but for a change in the absolute motion of the objects with respect to absolute space. The fact that it alters the dynamical law is therefore not surprising (and is not a violation of the principle). This version of the argument is a straightforward application of the symmetry principle.

Wigner ([1964]) has described electromagnetic gauge symmetry as a similar connection between change of representation and the dynamical law: 'two different descriptions of the same situation should develop, in the course of time, into two descriptions that also describe the same physical situation'. Wigner's view, according to Martin ([2003]), significantly contributed to the modern textbook presentation of the gauge principle. Nakahara ([2003]), for example, formulates it simply as 'physics should not depend on how we describe it'.

In order to formulate a general principle, I generalize from Earman's principle for spacetime symmetries using the concepts of kinematics and dynamics. The term

[2] An extensive discussion of spacetime symmetries and their relation to spacetime structure is given by Pooley ([2013]). For simplicity, I focus below on the spatial part of the transformations and avoid the temporal part whenever possible.

kinematics is used here in a broad sense to refer to the representation of the possible states of a system according to a given physical theory. A kinematical symmetry is an automorphism of the kinematical structure.[3] Kinematical symmetries are therefore passive transformations, a change in the mathematical labels of the different physical states postulated by the theory. In contrast, dynamical symmetries are transformations of the values of the dynamical variables, under which the form of the dynamical law remains invariant.

Using the above definitions, I formulate a general symmetry principle.

> Symmetry Principle: Every kinematical symmetry of a theory should also be a dynamical symmetry.

The above passive version of Newton's bucket argument, Einstein's general principle of relativity, and the textbook version of the gauge argument can all be regarded as expressions of this principle.

2.2. A methodological principle

There are different ways to apply the above symmetry principle. In this section, I formulate a methodological principle that supports the construction of theories that satisfy it. I start by describing Mach's analysis of Newton's bucket experiment in terms of transformations and symmetries, and generalize from this example.

The passive version of Newton's bucket argument given in the previous section is based on a comparison of two descriptions of one situation. The original bucket argument by Newton, in contrast, was based on a comparison of two different physical situations: a vessel of water at rest versus the vessel after it has gradually been rotated from rest to a state of uniform angular velocity. The argument against relationism is based on noting that in both cases there is no relative motion between the water and the bucket, while the shape of the surface of the water turns out to be different. The outcome is thus not determined by the relative motions between the components of the system. It has to be determined by the motion with respect to something else, which Newton believes to be absolute space.

Mach's ([1919], p. 232) famous reply was that the bucket experiment 'simply informs us, that the relative rotation of the water with respect to the sides of the vessel produces no noticeable centrifugal forces, but that such forces are produced by its relative rotation with respect to the mass of the earth and the other celestial bodies'. He therefore suggests to replace Newton's laws of motion with modified laws that would be constructed to reflect this postulated origin of centrifugal forces.

According to the Machian view, the transformation in Equation (1) is, indeed, no more than a change of representation. The reason that it is not a symmetry of

[3] This definition is a special case of the definition given by Redhead ([2003]) to passive symmetries, and is also close to Healey's ([2009]) definition of 'theoretical symmetries'.

Newtonian mechanics is that the latter theory is an approximate description of small subsystems of the universe. Thus, the description of the bucket and the water in terms of Newtonian mechanics does not take into account the influence of some relevant objects (the celestial bodies). The full symmetry would only be revealed by the more fundamental dynamical law, which would take into account all of the relevant degrees of freedoms.

In order to characterize Mach's suggestion in terms of transformations, we shall have to consider active transformations. The basic definition of active transformations is transformations that map the set of possible states to itself, thus replacing one physical configuration with a different configuration (and are therefore, generally, non-symmetries).[4]

Newtonian mechanics is not invariant under passive rotation. In the transition from describing a mechanical system using one spatial frame of reference to a description using a second frame of reference that is rotating with respect to the first, the form of the equations of motions change due to pseudo-force terms. The Machian step postulates an interaction between the described system and external objects; when the system is actively rotated with respect to the external objects, the system experiences an actual force, identical in its form to the pseudo-force in the passive case.

Obviously, there is no *a priori* guarantee that the world would behave this way. In Mach's case, he did not even formulate a theory of such interaction. What he did show is that modifying the dynamical law in this way, would result in a truly relationist theory, a theory in which Equation (1) is indeed a symmetry transformation. In other words, we start from a theory that violates the symmetry principle, and construct a theory that satisfies it by postulating the existence of active transformations that alter the dynamics in a particular way.

The active transformation, in this case, is not a change in the state of the original system, but rather a change in the state of the entire universe. More precisely, a change in the relations between the system (the bucket and the water) and external objects (the celestial bodies). The relevant active transformations are therefore only defined given the identification of a 'system' and some 'external object'. Such a partition is natural in the case of transition from Newtonian mechanics of localized mechanical systems to a possible Machian theory of the universe. The crux here is the postulated correspondence of every passive transformation of the system, to an active transformation of the system with respect to the external object.

[4] A major source of the common confusion between passive and active transformations is that in many cases active transformations take the same mathematical form as passive ones. Furthermore, in some cases it would be a matter of interpretation whether a given transformation of the variables is active or passive. For example, someone who believes in the existence of Newtonian absolute space would interpret a fixed translation $\vec{r} \to \vec{r} + \vec{R}_0$ as an active transformation of all material objects with respect to absolute space (every object is now in a different part of space, therefore the physical state has changed). But if absolute space does not exist, this transformation is just a passive transformation of the coordinates. In many other cases, the transformation would be observable and its active nature would be apparent. A simple example is the translation of a physical system with respect to other material objects.

Provided the empirical success of Newton's laws of motion, and that their spatial symmetry group is the Galilei group in Equation (2), we have encountered two strategies for satisfying the symmetry principle. The first is the introduction of unobservable Newtonian absolute space, thus reducing the kinematical symmetry group to the desired group. The second is the Machian step that extends the dynamical symmetry group by introducing additional degrees of freedom. Clearly, the latter strategy is the one that is more likely to yield empirically testable predictions.[5] The Machian step and the gauge argument, we shall claim, both apply the following methodological principle, relevant when a theory successfully describes a given physical system, but does not satisfy the symmetry principle.

> Methodological Principle: For every kinematical transformation of the system, postulate the existence of an active transformation of the system with respect to an external object, that induces a change of an identical form in the dynamics.

This methodological step is an amplification of the scope of the theory: from a theory that describes a given system, to a broader theory that describes the coupling of that system to something external.

This methodology is somewhat different from Mach's original intention, and also from various other meanings that are attributed to the term Mach's principle (see Barbour and Pfister [1995], p. 530). In particular, the holism that exists in a Machian universe is not a necessary feature of relational theories, nor is it necessary for my purposes. It is enough that the relations between parts of a system and some external object are physically relevant.

Any successful application of the methodological principle would have to take empirical input into account. The methodological principle conjectures the existence of relevant physical degrees of freedom that the original theory did not take into account. Empirical knowledge of the world is then required in order to associate these theoretical additional degrees of freedom with actual physical objects (such as celestial bodies). The new theory that describes the coupling of the original system to these objects can then be put to further empirical tests.

Note that even when a given theory does not satisfy the symmetry principle, it is possible that some kinematical symmetries would happen to be dynamical symmetries as well. The corresponding active transformations would therefore induce no change in the dynamical law. These transformations change the state of the given system with respect to its environment, while at the same time preserving the dynamics of the system. I shall refer to these symmetries as active symmetries (see a similar definition in Brading and Brown [2004]). For example, passive Galilean boosts in Equation (2) are dynamical symmetries of Newtonian mechanics. The corresponding active transformations put a given system in motion with respect to its

[5] Symmetry considerations were indeed useful for the construction of Machian theories of mechanics. The most famous ones are those by Barbour and Bertotti ([1977], [1982]); see also (Huggett and Hoefer [2018], Section 8.2) for a brief discussion of their empirical testing.

environment as described by the famous ship experiment (Galilei [1967]). In this example the transformed system is a ship, first at rest with respect to the shore, and then sailing at a constant velocity. This transformation is a symmetry since the dynamics of the ship (describing the motion of objects on board with respect to the ship) is not affected by the change.

2.3. A ghost in classical mechanics

The controversy regarding the nature of space shows that classical mechanics is inherently haunted by a ghost of the same kind as described by Wigner. Here the ghost is the frame of reference. It does not correspond to a directly observable physical object, and it can be moved around without changing the physical situation. Newton's substantivalist way of dealing with the ghost is to let it take on a life of its own, in the form of absolute space. In contrast, Leibnizian relationism attempts to exorcise the ghost, if not from the formalism then at least from the ontology. It is very likely that this motivation was shared by Mach when he suggested that accelerations are defined with respect to all the masses in the universe.

The suggested understanding of Mach's principle as an instance of the presented methodological principle is somewhat different. The presence of the ghost in the Newtonian formalism in an indication of the existence of a relevant physical object ignored by the theory. Instead of eliminating the ghost, Machian thinking first lets it take on a life of its own, this time in the form of an observable physical object with dynamical properties of its own. This object, the 'celestial bodies', defines the frames of reference in which Newton's laws hold (at least as an approximation).

It is important to note that sometimes a frame of reference is just a frame of reference, a formal auxiliary component and nothing more. A theory that employs such a frame of reference would essentially have kinematical symmetries, which reflect the equivalence of different choices of frame of reference. In this case, the kinematical symmetries would all correspond to dynamical symmetries, and the symmetry principle would be satisfied. This is an indication that the ghost is harmless, and can be let alone.

3. A Toy Theory

Newtonian mechanics does not satisfy the symmetry principle presented in Section 2. I begin this section by presenting a simple toy theory that does. The theory is inspired by the 'minimal prototype of a gauge theory' by Rovelli ([2014]) and closely resembles it. I will then show how the symmetry principle together with the methodological principle can help physicists, who live in the universe described by the theory but only have partial knowledge of it, discover this theory.

Consider a one-dimensional universe in which $N + 1$ point particles are moving and interacting with each other. Let us assume that it is a truly relationist universe:

the dynamical variables are relations a_{ij}, each corresponding to a pair of particles. Their possible values define the configuration space. Their temporal derivatives are denoted $\dot{a}_{ij}$. In this simple one-dimensional universe, it is also possible to choose $N + 1$ real position variables x_i such that for all i and j we get $a_{ij} = x_i - x_j$.

Kinetic energy here would be defined for every pair of particles. For the i-th and j-th particles, with masses of m_i and m_j, the kinetic energy is:

$$T_{ij}(\dot{a}_{ij}) = \frac{1}{2}\frac{m_i m_j}{m}\dot{a}_{ij}^2, \tag{3}$$

with m denoting a universal mass constant. This kind of kinetic energy deserves to be called an interaction, as it is not inherently different from the way particles interact through potential energy that depends on their relative distance $V(a_{ij})$. For now, I also assume that the masses of all the particles are identical $m_i = m$ for all i. The Lagrangian is therefore obtained by summing over the pairs of particles:

$$\mathcal{L}_0 = \sum_{j<i}\left[\frac{1}{2}m\dot{a}_{ij}^2 - V(a_{ij})\right] \tag{4}$$

$$= \sum_{j<i}\left[\frac{1}{2}m(\dot{x}_i - \dot{x}_j)^2 - V(x_i - x_j)\right]. \tag{5}$$

The variables x_i correspond to the positions of the particles in relation to an arbitrary frame of reference. Unlike in Newtonian mechanics, this frame of reference does not provide an absolute notion of acceleration. The Lagrangian in Equation (5) is invariant under arbitrary time-dependent translation of the frame of reference:

$$x_i \rightarrow x_i - \lambda(t). \tag{6}$$

Clearly, this symmetry does not describe a property of the world. It originates in the way the world is represented in the theory using an artificial frame of reference, a ghost in the kinematics with no dynamical role. This symmetry merely expresses the trivial equivalence of Equation (4) and Equation (5), the possibility of eliminating some of the mathematical structure of Equation (5). It is a kinematical symmetry, as well as a symmetry of the dynamical law. The symmetry principle is thus satisfied. The symmetry exists because of the way the system is represented. This choice of representation is no more than a matter of convention and convenience. No physical object is hiding behind this ghost, and no new physical knowledge can be obtained through it.

The equations of motion derived from Equation (5) are:

$$\ddot{x}_i - \frac{1}{N}\sum_{j\neq i}\ddot{x}_j = \frac{1}{Nm}\frac{\partial}{\partial x_i}\sum_{j\neq i}V(x_i - x_j). \tag{7}$$

Unlike the Newtonian absolute notion of acceleration, the acceleration of a particle in this theory is measured in relation to the centre of mass of all other particles. This point, together with the invariance in Equation (6), distinguishes this theory from Newtonian mechanics.

Nevertheless, the physics of some subsystems in this universe could be approximately Newtonian. To see this let us add an object of mass M that is external to the system. This object can be thought of as a representation of the totality of 'celestial bodies' surrounding the system, or just a relevant external object that interacts with the system's particles. The relation $c_i \equiv x_i - X$ denotes the spatial relation between the i-th particle and the external object. For simplicity, I assume that there is no interaction between the particles and the external object other then the kinetic coupling. The new Lagrangian is therefore:

$$\mathcal{L}_1 = \mathcal{L}_0 + \sum_i \left[\frac{1}{2} M \dot{c}_i^2\right] = \sum_{j<i} \left[\frac{1}{2} m(\dot{x}_i - \dot{x}_j)^2 - V_{ij}(x_i - x_j)\right] + \sum_i \left[\frac{1}{2} M\left(\dot{x}_i - \dot{X}\right)^2\right]. \tag{8}$$

The equation of motion for x_i is similar to Equation (7), but the acceleration is now measured in relation to the new centre of mass:

$$\ddot{x}_i - \frac{\sum_{j\neq i} m\ddot{x}_j + M\ddot{X}}{M_0} = \frac{1}{M_0}\frac{\partial}{\partial x_i}\sum_{j\neq i} V(x_i - x_j), \tag{9}$$

with $M_0 = Nm + M$.

To see the relevance of this example, let us take the perspective of physicists who live and conduct their experiments within the $N + 1$ particle system. They may be unaware of the existence of any external object, or of its coupling with the system, so the variable X will not appear in their theories. They would find that the law of motion takes a simple form in some preferred frames of reference (we know that these are the ones in which $\ddot{X} = 0$). In these frames of reference the physicists would find that their observations are explained by the law:

$$\ddot{x}_i - \frac{\sum_{j\neq i} m\ddot{x}_j}{M_0} = \frac{1}{M_0}\frac{\partial}{\partial x_i}\sum_{j\neq i} V(x_i - x_j). \tag{10}$$

(If the second term is negligible this law becomes completely Newtonian. If some accelerations are sufficiently large, however, the second term may become significant and the physicists could discover it with their experiments.)

While the equations of motion in Equation (10) only contain the variables $\{x_i\}$, they are not invariant under the general time-dependent transformation of the variables in Equation (6). The kinematical symmetry is now not a dynamical symmetry. In a general frame of reference an extra term has to be added, the Machian version of a pseudo-force. (From the way Equation (9) transforms under Equation (6) it can be obtained that this force is equal to $-\frac{M}{M_0}\ddot{\lambda}$, where $\lambda(t)$ defines the transformation from a reference frame in which Equation (10) holds.)

The ghost frame of reference that haunts the theory in Equation (10) is thus more than a matter of harmless convention. The fact that the dynamical symmetries are a small subset of the kinematical symmetries is the indication of that. This violation of the symmetry principles is the starting point of the methodology suggested here. Its goal is to replace the theory that does not satisfy the symmetry principles with a new theory that does satisfy them and also has greater predictive power.

At this point, the physicists who apply Equation (10) can follow the methodological principle. For every passive transformation of the form in Equation (6), they postulate an active transformation of the same form. It is easy to see that the active transformation does not change the internal relations that describe the system. The change that is induced by this active transformation must be a change in the state of the system with respect to an external object. In the simplest case the external object would correspond to a single dynamical variable X. The transformation in Equation (6) is an active transformation of the system with respect to the external object if the positions x_k of the particles with respect to the arbitrary frame of reference are replaced by their position with respect to the external object:

$$x_k \rightarrow x_k - X \tag{11}$$

Indeed, this step generates (after some algebra) the correct equations of motion in Equation (9). This method therefore allows the physicists to obtain the law for the interaction of the particles that can be directly observed, with an external object.

In order to formulate a complete theory, observations and experiments are still required. Observations are required in order to associate the theoretical degree of freedom X with some physical objects. Experiments would allow measurement of the mass M.

Finally, I note that the passive transformation for the extended theory is an extension of Equation (6) that is applied to both the system and the external object, and maintains the relations between them:

$$x_i \rightarrow x_i - \lambda(t), \qquad X \rightarrow X - \lambda(t). \tag{12}$$

This example is to show that if the universe were simple and Machian, then applying Mach's principle would closely resemble the way the gauge principle is applied. In both cases, an interaction-free theory whose dynamical symmetry group is smaller than the kinematical symmetry group is extended to take an additional physical object into account, together with its dynamical properties. The extended theory has an extended symmetry that is now both kinematical and dynamical.

4. Quantum Theory and Gauge Symmetries

4.1. The representations of a quantum system

Let us consider a quantum system that remains in a pure state at all times. Quantum theory postulates that the possible states of the system correspond to points in a projective Hilbert space. This state space can be represented using different bases of the Hilbert space. For every two bases there is a unique unitary transformation that connects them.

Given an orthonormal basis $\{|\phi_j\rangle\}$ of the Hilbert space, each state $|\psi\rangle$ can be characterized by the coefficients $\{c_j\}$ of the expansion $|\psi\rangle = \Sigma_j c_j|\phi_j\rangle$. The unitary transformation U_{ba} defines a passive transformation from a representation in one basis $\{|\phi_j^{(a)}\rangle\}$ to a representation in another basis $\{|\phi_j^{(b)}\rangle\}$ in the sense that it defines a transformation matrix $T(U_{ab})$ that transforms the values of the coefficient used to represent any state in the first basis to the coefficients that would be used to represent the same state in the second basis. The matrix elements are given by $T_{jk}(U_{ba}) \equiv \langle\phi_j^{(a)}|U_{ba}^\dagger|\phi_k^{(a)}\rangle = \langle\phi_j^{(b)}|\phi_k^{(a)}\rangle$ and the transformation is:

$$c_j^{(b)} = \sum_k T_{jk}(U_{ba})c_k^{(a)}. \tag{13}$$

The bases used to represent a system are not internal to it. A physically meaningful basis consists of the eigenstates of an Hermitian operator that represents an actual measurement that can be performed on a system by an external apparatus. When we say, for example, that the spin state of an electron is $|\uparrow\rangle$, we are making a statement about the relation between the direction of the spin of the electron and an external physical object that defines the z-axis.

Any operator $\hat{B}$ can be represented in terms of the basis states: $\hat{B} = \Sigma_{jk} b_{jk}|\phi_j\rangle\langle\phi_k|$. A full passive transformation of the representation of the quantum state together with the state of the reference objects is a change of representation $|\phi_j\rangle \to U|\phi_j\rangle$ and $\langle\phi_k| \to \langle\phi_k|U^\dagger$ that is applied to all appearances of the basis states. As a direct consequence, all quantum states and operators transform:

$$|\psi\rangle \to U|\psi\rangle, \qquad \hat{B} \to U\hat{B}U^\dagger. \tag{14}$$

Any unitary transformation, U, also defines an active transformation that changes the state of a given system: $|\psi\rangle \to U|\psi\rangle$, an unitary transformation that is applied to a given state. It represents an active change in the physical state of the system in relation to the reference that defines the operators. This transformation can be equivalently written in the Heisenberg picture as a transformation $\hat{B} \to U\hat{B}U^\dagger$ of the operators, rather than of the state $|\psi\rangle$.

4.2. Relative variables: quantum phases

In the example given in Section 3 it is manifestly clear how the relational nature of the variables gives rise to the multitude of representations and therefore to symmetry transformations. The goal of this section is to show how the multitude of representations of quantum systems and the transformations between them that were presented in the previous section also manifest the relational nature of fundamental physical variables: quantum phases.

To begin with, note that it is the relational nature of quantum phases that distinguishes them from any classical analogue. A classical sinusoidal travelling wave on a string is described by the function: $\psi_1(x, t) = A \sin(k(x - ct) + \varphi_1)$. Once a particular point is marked as the origin, and a particular moment of time is denoted $t = 0$, the phase φ is well defined (up to addition of an integer number of 2π). An interference pattern would appear when this wave encounters a second wave: $\psi_2(x, t) = A \sin(-k(x + ct) + \varphi_2)$. The phase difference $\varphi_1 - \varphi_2$ determines the resultant pattern (which points on the string would be nodes and anti-nodes). This phase difference however, does not represent a fundamental relation: each of the two phase factors φ_1 and φ_2 represents an independent quantity that can be understood as a relation between the wave on the string and the spatio-temporal frame of reference. The phase factor φ_1, for example, could have been measured from the form of the corresponding travelling wave even in the case where it is the only wave on the string, and there is no interference at all. The same holds for the classical two-dimensional analogue, the double slit interference experiment, in which the wave from each source (or slit) can be described by a phase factor that is measurable prior to any interference.

The case is very different in quantum mechanics. While quantum phases are responsible for the wave-like behaviour of quantum particles in interference experiments, the absolute phase attributed to an individual wave-packet has no meaning in the theory and cannot be measured. If in a quantum double-slit experiment, for example, only one slit is open, then it is meaningless to talk about the value of the phase of the wave-function that propagates though the slit, and it is impossible to measure it. The relative phase between the two slits makes a difference only once interference has taken place. That is why it is justified to say that the phase is a fundamental relation that describes the quantum particle. It is an internal relation between the different components of the wave function, not a relation between a particular component and some reference frame imposed by an external observer.

Nevertheless, we commonly find it more convenient to refer to quantum phase relations as differences between two phase factors, represented by numbers on the real axis. But in the quantum case this convention should be seen as a matter of mere convenience. This is reflected by the phase shift symmetry of the theory. Namely, the quantum states $|\phi\rangle$ and $e^{i\varphi}|\phi\rangle$ are commonly assumed to represent the same physical state for all values of φ.

Each physical state therefore has infinitely many (normalized) representations because the formalism uses absolute values to represent a relational property. This seems similar to the classical example of Section 3. The interesting difference stems from the quantum notion of superposition of basis-states. What do we get if we apply the transformation $|\phi\rangle \rightarrow e^{i\varphi}|\phi\rangle$ to the basis-states of a quantum system? The answer is that we get no more and no less than the unitary transformations described in the previous section.

Given an arbitrary basis $\{|\phi_j\rangle\}$, it is easy to see that a phase transformation of the basis-states,

$$|\psi\rangle = \sum_j c_j|\phi_j\rangle \to \sum_j e^{i\varphi_j} c_j|\phi_j\rangle \equiv U|\psi\rangle, \tag{15}$$

is, by definition, unitary ($U^\dagger U = 1$). Furthermore, for every unitary transformation U there is a basis $\{|\phi_j^{(U)}\rangle\}_j$ in which it is diagonal and takes the form of Equation (15). In terms of passive transformations, that means that every change of representation (from one basis to another) can be described as phase transformation of the basis-states of a particular basis.

Yet, quantum phases are meaningful physical variables. The two superpositions $|\Psi_A\rangle = c_1|\psi_1\rangle + e^{i\varphi_A} c_2|\psi_2\rangle$ and $|\Psi_B\rangle = c_1|\psi_1\rangle + e^{i\varphi_B} c_2|\psi_2\rangle$ (with complex coefficients c_1, c_2 and real $\varphi_A \neq \varphi_B + 2\pi n$) represent different physical states. A temporal evolution in which $|\Psi_A\rangle$ is transformed into $|\Psi_B\rangle$ changes the observed properties of the system. This change is described by the theory with respect to some external reference. The external reference defines the meaning of the operators by identifying the basis-states with elements that are external to the system, such as points on a screen or the direction of an external field. The unitary temporal evolution of a quantum system can always be described as such an active transformation—a shift between the relative components of the state in some preferred basis. This basis consists, of course, of the eigenstates of the Hamiltonian.

4.3. Gauge transformations

Unitary transformations induce relative phases between the states of a particular basis. If this basis is the position-basis, the transformation is a gauge transformation. This understanding of gauge transformations together with the principles and methodology presented in Section 2 will be used in this section to understand the simplest example of a gauge theory: the coupling of a quantum particle to a classical electromagnetic field.

Consider a quantum particle whose state is described by a wave function $\psi(\vec{x}, t) = \langle \vec{x}|\psi(t)\rangle$ (ignoring the spin degree of freedom). A transformation of the form in Equation (14) that is diagonal in position-basis is a passive local phase transformation. The transformation is a formal replacement of the position-basis eigenstates with eigenstates that only differ by phase defined by an arbitrary smooth function $\Lambda(\vec{x}, t)$:

$$|\vec{x}\rangle \to |\vec{x}'\rangle = U|\vec{x}\rangle = e^{i\Lambda(x,t)}|\vec{x}\rangle. \tag{16}$$

The observable quantities remain unchanged as long as the replacement is consistently applied to all appearances of the basis states. The values of the wave function do change, as well as the form of the operators, in particular the momentum operators:

$$\psi(\vec{x},t) \rightarrow \psi'(\vec{x},t) = \langle \vec{x}'|\psi(t)\rangle = e^{-i\Lambda(\vec{x},t)}\psi(\vec{x},t), \tag{17}$$

$$p_i \rightarrow p_i' = Up_iU^\dagger = p_i - \omega_i(\vec{x},t), \tag{18}$$

with $\omega_i(\vec{x},t) = \hbar\frac{\partial}{\partial x_i}\Lambda(\vec{x},t)$. This transformation of the momentum operators is not an additional requirement. It expresses the same replacement of basis states, and is obtained by a straightforward calculation using Equation (14).

Hence, when it comes to local phase transformations, a significant physical variable does depend on the choice of representation: the momentum operator. Dirac had noticed this and pointed out that 'by a suitable change in the phase factors, the function [Λ][. . .] can be made to vanish and [the] equations [$p_i = -i\hbar\partial/\partial x_i$] are made to hold' (Dirac [1958], p. 93, brackets indicate adjustment of notation). While this is true from the point of view of anyone whose primary interest is to obtain predictions from the Schrödinger equation, it is an important clue for the right way of extending the scope of the theory. The way the momentum operator transforms under the kinematical symmetry in Equation (16) indicates that this transformation is not a dynamical symmetry. The Schrödinger equation changes its form:

$$i\hbar\frac{\partial}{\partial t}\psi(\vec{x},t) = \frac{\sum_i p_i^2}{2m}\psi(\vec{x},t) \rightarrow i\hbar\frac{\partial}{\partial t}\psi(\vec{x},t) = \frac{\sum_i(p_i - \omega_i(\vec{x},t))^2}{2m}\psi(\vec{x},t). \tag{19}$$

According to the methodological principle, an extended invariant theory may be constructed in which a transformation of the same form would describe an active change in the state of the particle with respect to some external degrees of freedom. In the simplest case these additional degrees of freedom constitute a classical field. A transformation of the form in Equation (18) can be regarded as an active transformation with respect to the field if the field couples to the momentum operators. This coupling is achieved by the substitution:

$$p_i \rightarrow p_i - \omega_i(\vec{x},t). \tag{20}$$

This expression should not be confused with the transformation in Equation (18). The ω's that appear in the two expressions, as well as the arrows represent very different things. In Equation (18), we simply express the momentum operator in a different basis of the Hilbert space. The notation p' stands for the same momentum operator of the system, which is expressed in the RHS of Equation (18) in terms of its effect on the 'old' basis states. Its components therefore satisfy the known momentum commutation relations: $[p_i', p_j'] = 0$. These commutation relation hold under the condition: $\frac{\partial\omega_i}{\partial x_j} - \frac{\partial\omega_j}{\partial x_i} = 0$ (see Bohm *et al.* [2013], pp. 22–3). This condition is automatically satisfied by the definition $\omega_i(\vec{x},t) = \hbar\frac{\partial}{\partial x_i}\Lambda(\vec{x},t)$.

The transformation in Equation (20), in contrast, means that in order to understand the way the particle may interact with other physical objects, I attempt to extend this theory by replacing the appearance of the momentum in the dynamical law with a relation between the momentum and an external field. In this case, the success of the attempt is not guaranteed by the mathematics; it has to be verified by experiment.

In order to do so, field ω has to be recognized as some particular physical field. Indeed, if we identify field ω as proportional to magnetic vector potential $\vec{A}$, and use the charge of particle q as a coupling constant such that $\vec{\omega} = \frac{q}{c}\vec{A}$, then we get the well-established Schrödinger equation for a charged particle under electromagnetic influence. The extended particle + potential system is invariant under the passive gauge transformation in Equations (16)–(18) when it is accompanied by $\frac{q}{c}\vec{A} \rightarrow \frac{q}{c}\vec{A} + \hbar\nabla\Lambda$. The replacement of the gauge-dependent momentum variable with an invariant variable representing the relation between the system and the field thus extends the kinematical symmetry of the particle to the larger particle and field system, making it a dynamical symmetry as well.

In this case, the components of the momentum operator of the particle $\vec{p}$ commute, but there is no *a priori* reason to assume that so would the components of the combination $p_i - \omega_i(\vec{x}, t)$. They don't commute in any point of space in which we have a magnetic field. Furthermore, the identity $\omega_i(\vec{x}, t) = \hbar\frac{\partial}{\partial x_i}\Lambda(\vec{x}, t)$, which in the passive case is satisfied by definition (since we begin with a well-defined single-valued transformation), can no longer be met in the presence of a magnetic flux. The immediate consequence is the appearance of measurable non-integrable phase factors.

With this understanding of gauge, the meaningful physical variable appears to be $(\vec{p} - \frac{q}{c}\vec{A})$, which is interpreted as the relation between the momentum of the particle and the field. The interpretation of this relation as a physical quantity opens the door to an interpretation of electromagnetism that differs from the common approaches (Belot [1998]). The dependence of the coupling of the particle and the field on this relation is analogous to the way the inertia in the example given in Section 3 depends on the relation $(x_i - X)$.

Due to the relational nature of quantum phases, the choice of a spatial frame of reference only determines position-basis-states up to a phase factor at each point. There are therefore infinitely many ways to define a basis in which the state of the particle can be represented. The law governing the dynamics of the particle under the external influence is obtained by replacing the momentum operator with the relation between the momentum and an external field. This method allows us to correctly guess the form of the dynamical law that describes the effect of the interaction on the particle.

5. The Gauge Argument

5.1. The gauge argument: what makes it work?

The formal replacement, $\partial_\mu \rightarrow D_\mu \equiv \partial_\mu + i\frac{q}{\hbar c}A_\mu$, of partial derivatives with gauge-covariant derivatives is known to successfully introduce the electromagnetic influence into the free particle Schrödinger equation $i\hbar\partial_t\psi = -\frac{\hbar^2}{2m}\nabla^2\psi$, as well as into the Dirac equation for a spin-half particle $(i\hbar\gamma^{\mu}\partial_{\mu} - mc)\Psi(x) = 0$. Textbooks attribute this replacement to the requirement of local gauge covariance, the requirement that the global phase transformation remains a symmetry of the dynamics when it is replaced by a local (coordinate-dependent) transformation. This is the simplest application of the gauge principle, first applied by Yang and Mills ([1954]), and by now well known for its major role in the derivation of the dynamics of elementary interactions and the properties of force carriers.

The gauge principle is commonly justified by the freedom to change convention or a frame of reference. The roots of this view go back to Wigner ([1964], [1967]; see Martin [2003]), who presented gauge transformations as a passive invariance property of some particular dynamical laws of interaction (in contrast to the geometric symmetries that hold for relations between events). A gauge field that is introduced in this manner takes the formal role of a connection on a principal bundle representing the internal degree of freedom over spacetime manifold. Namely, it is a mathematical structure that gives meaning to the notion of 'the same phase' in different locations.

Teller ([1997]) noted the mystery posed by the gap between the understanding of gauge as a matter of labelling, linguistic conventions, and its 'dramatic repercussions' as a concept that imposes the introduction of new physical fields into the equations. Furthermore, Brown ([1999]) noted that the requirement for local gauge covariance can only explain a flat connection. It cannot explain the interaction that acts on the particle through the curvature. Neither can it account for the other direction of the interaction, namely the action of the particle on the field. Lyre ([2000]) extended this criticism: 'in order to obtain the full Dirac–Maxwell theory we need a truly physical principle. Otherwise, there remains a "missing link"—at least from the foundational point of view'. Without an additional physical principle, there is no reason for identifying the connection that appears in the gauge covariant derivative with a particular physical field, nor can the requirement for gauge covariance dictate the existence of such a field (Healey [2007]). With no such principle available, the best way to understand the gauge principle is heuristically (Martin [2002]).

This article attempts to address these worries on two levels. The first is an account of the methodology. The gauge argument, as presented here, does set off from the view of gauge transformations as a change of description that transforms one quantum-theoretical representation of a physical state into another representation. This passive view motivates the attempt to look for a dynamical law that is invariant under the transformations, and to modify existing theories whose laws are not gauge-invariant. The gauge argument is thus described as a combination of theoretical symmetry principle and a methodological step motivated by it.

As long as no reference is made to some contingent property of the world, the account is not sufficient for resolving the aforementioned worries. On the second level, therefore, I claim that the success of the gauge argument is anchored in the relational nature of the physical quantities described by gauge-dependent variables.

This physical view of gauge is based on the account by Rovelli ([2014]): 'Gauge invariance is not just mathematical redundancy; it is an indication of the relational character of fundamental observables in physics. These do not refer to properties of a single entity. They refer to relational properties between entities: relative velocity, relative localization, relative orientation in internal space, and so on'. This relational nature is exploited by the methodological principle suggested here, that is based on active transformations that change the relation between the system and an external object. The original interaction-free theory is expressed in terms of gauge dependent dynamical variables that represent relations between a physical field and a mathematical connection, a generalization of the concept of a frame of reference. The methodological step replaces these variables with gauge invariant variables that are naturally interpreted as relations between two physical fields. The result is a theory whose dynamics and empirical content are gauge invariant; the symmetry principle that was violated by the interaction-free theory, is now satisfied.

To understand the gauge principle in this light, we must distinguish between two distinct questions: (i) Why is the theory of interacting fields invariant under certain transformations, whereas the theory of the free field is not? (ii) Why are the relevant transformations local (coordinate dependent)?

Our answer to the first question is that a theory that disregards relations between the system it describes and other relevant physical objects cannot reveal a complete picture of the symmetries. In a Machian universe, the invariance of the laws to passive rotations is only revealed in the theory of the whole universe, since the relations between all pairs of objects are relevant. The physical properties of a field depend on its relations to other fields. A gauge transformation that acts on two fields, such as

$$\psi(x^{\mu}) \to e^{i\Lambda(x^{\mu})}\psi(x^{\mu}), A_{\mu} \to A_{\mu} + \frac{\hbar c}{q}\partial_{\mu}\Lambda, \tag{21}$$

is a passive transformation of the coupled system. It acts on both fields and maintains the physical relations between them (similar to the transformation in Equation (12) on the system itself and the object to which it is coupled). In contrast, the transformation $\psi(x^{\mu}) \to e^{i\Lambda(x^{\mu})}\psi(x^{\mu})$ (or an equivalent transformation of the operators in the Heisenberg picture) actively changes the relation between ψ and A_{μ} and is therefore not a symmetry transformation of the theory of ψ. In the same way, Equation (6) is not a symmetry of Equation (8), and an active rotation of the bucket and the water is not a symmetry transformation in Newtonian mechanics.

As for the second question, this account suggests that the relevant symmetry is local due to the quantum structure of the world. Quantum theory identifies different locations as different components of the superposition. The relational nature of the phase is expressed in the assumption that $|\phi\rangle$ and $e^{i\varphi}|\phi\rangle$ can represent the same physical state. The assumption is valid regardless of whether the actual state is $|\phi\rangle$ or a superposition of $|\phi\rangle$ and other states. Eigenstates of position are no exception. Indeed, the relevance of the quantum structure of the world to gauge theories was already recognized when both quantum mechanics and the very idea of gauge were newly born (Weyl [1929]; London [1997]).[6]

Formally, a passive local phase transformation is a change of the connection form of the principal $U(1)$ bundle over spacetime manifold. The methodological principle implies that every such transformation is postulated to correspond to an active transformation of the state with respect to an external physical field. The entity with respect to which phases are shifted is now a physical field, instead of a mathematical connection. Passive phase transformations cannot change gauge invariant quantities that characterize the bundle, such as local curvature and holonomies. The possible active transformations, in contrast, are not restricted in this way as the state of the field is arbitrary and dynamically changing. There are therefore active transformations that have no passive parallel. They reflect new phenomena generated by the interaction, ones that cannot be described by the interaction-free theory. These phenomena, as was noted by Brown ([1999]), are indeed not a consequence of the requirement for local gauge covariance. The role of this requirement is methodological: it guides the construction of the correct mathematical object (connection) that can be generalized to a mathematical description of the field that describes the interaction.

5.2. Interpretation and observability of gauge symmetries

The question of whether gauge symmetry transformations can be observed has been debated among several thinkers (Kosso [2000]; Brading and Brown [2004]; Healey [2009]; Greaves and Wallace [2014]; Friederich [2014]). The question, formulated by Kosso, concerns direct observability. For an experiment to be considered a direct observation of a symmetry transformation, two facts have to be independently verified empirically. First, we must verify that the given transformation has indeed taken place, that something has changed in the world. Second, we must also verify that the transformation is an invariance, meaning that the same laws apply to the new situation. In large part, the debate revolves around the question, also discussed below, of whether the version of the double-slit experiment presented by 't Hooft ([1980]) can be regarded as an observation of gauge symmetry.

Kosso's definition means that as long as gauge transformations are regarded as no more than passive changes of the mathematical representation, it is meaningless to discuss direct observations of them. Observability requires a notion of an active change in the world.

In the different examples discussed in this article, there are several cases in which a passive transformation has a parallel active transformation. For example, it is agreed

[6] The main reason that the idea of gauge is now considered distinct from quantum theory is probably its applicability to interactions between classical fields. Yet, this applicability can be due to the underlying quantum nature of the physical objects that the classical fields represent.

that Galileo's ship experiment is the active parallel of a passive uniform boost. But what exactly does that mean?

My answer to this question is different from the common approach. I propose that a passive transformation of a theory that describes a certain physical situation can only correspond to an active transformation in a theory that describes a different, extended, situation. I start from a passive transformation applied within a theory that describes a given system that does not interact with anything external. The theory in Equation (5) of the $N + 1$ particles is one example, and the Schrödinger theory of an isolated quantum particle in a pure state is another. To obtain a theory with the corresponding active transformation, two steps must be taken. First, the existence of an external object is postulated. Second, the original system is represented in a frame of reference that is defined in some way by the system's relation to the external object. Thus a change in the relation between the external object and the system corresponds to a change of the frame of reference in which the original system is represented, and a correspondence is established between each passive symmetry (of the theory describing the system) and a particular active transformation (which may or may not be a symmetry of the theory that describes the system together with the external object). This definition requires some preferred isomorphism between the possible relations of the system and the external object, and the possible representations of the system. In the toy Machian theory described in Section 3, for example, this can be achieved by fixing the origin to the external object, so that the absolute coordinate of each of the particles of the original system reflects its distance from the external object.

Take, for example, a single-slit experiment in which single electrons go through a slit of narrow width and form a diffraction pattern on the other side. The propagation of the wave function through the slit is described by the Schrödinger equation. A global phase transformation is a kinematical symmetry, since it is a replacement of the same physical state (a point on the projective Hilbert space) with another representation of the same state. It is also a dynamical symmetry, as the Schrödinger equation is invariant under global phase transformations. The active version of this symmetry can be observed. To do this, we can open a second slit through which the wave function propagates, and use the wave packet coming through the second slit as an 'external object'.[7] In the region close enough to the first slit there is no overlap with the wave-packet propagating through the second slit, and it can be observed (through repeated experiments) that in this region inserting the phase shifter makes no observable change. But in a region further away from the two slits the form of the interference pattern between the two wave-packets depends on the presence of the phase shifter. Thus the phase shift is observable, and the invariance of the isolated wave to the phase shift is observable as well.

[7] As suggested by Greaves and Wallace ([2014]), but not as in the original suggestion of 't Hooft, ([1980]).

This analysis therefore leads to an agreement with Greaves and Wallace that this experiment can be regarded as a direct analogue of Galileo's ship. However, the symmetry that we claim that is observed is the active parallel of the global phase transformation symmetry, not the local one.[8] The phase shifter acts globally on the entire original 'system' (the single slit).[9]

Before I turn to consider the question of the observability of local phase transformations and gauge transformations, it is instructive to go back to the toy model presented in Section 3 and ask whether the symmetry of the model, the arbitrary change of frame of reference, is directly observable in the same sense that invariance to global phase transformation is observable through the double-slit experiment. The answer turns out to be negative. Equation (7) is the equation of motion of the system. It is invariant under an arbitrary change of frame of reference (6). Yet, as we have seen, when we add an external object, Equation (7) is no longer valid. The active parallel of Equation (6), in which the relation between the system and its environment changes, is not a symmetry transformation. For this reason, even after the physicists in the example have discovered the full, correct equation of motion of their universe in Equation (9), and even if they can directly measure the environment variable $X(t)$, there can be no direct observation of the symmetry. Indeed, the physicists would notice that all their observations can be accurately predicted with a law that is invariant under Equation (12). But despite this indirect evidence, there will be no 'Galilean ship' type of experiment for this symmetry. This should come as no surprise, since clearly, from the way the model was constructed, this symmetry is in itself not a property of the world, but only of the way it is represented.

The same model does contain a different symmetry transformation that can be observed. It is of course the Galilean boost $x \rightarrow x + vt$. Not only it is a kinematical and dynamical symmetry (since it is a special case of the general symmetry), but it is also an active symmetry. The internal dynamics of the system (Equation 10) does not change under the transformation $x_i \rightarrow x_i + vt$ that would be applied to all of the particles. The reason this transformation is a directly observable symmetry is simply that the equations of motion do not explicitly depend on velocity. Similarly, the global phase transformation can be observed in the double-slit experiment because the Schrödinger equation of a particle depends on the local phase gradient, not on the phase difference between spatially separated wave packets.

[8] Similarly, an analogous analysis would lead to the conclusion that Faraday's cage experiment should be regarded as a direct observation of global gauge transformation.

[9] Friederich ([2014]) provides a detailed account, similarly concluding that adopting the framework provided by Greaves and Wallace ([2014]) does not lead to their conclusion that local gauge symmetries are observable. According to Friederich, the phase shifter in 't Hooft's beam splitter experiment can be seen as changing the state of the environment, rather than the state of the wave-function that passes through the slit. Friederich's analysis thus shows that in the analysis of the experiment in Greaves's and Wallace's framework, it is underdetermined whether it is the state of the system that changes or the state of the environment. The claim above, stating that what actually changes is the relation between the system and the environment, naturally dissolves this underdetermination.

Local phase transformations are not observable symmetry transformations for the same reason that a non-Galilean boost is not an observable symmetry transformation in this toy theory. In both cases, the active parallel—the transformation that is applied to a subsystem—is generally not a symmetry transformation at all.

The relevant transformation that is a symmetry transformation is the full gauge transformation in Equation (21). None of the examples brought in the articles cited above can be regarded as an active version of this transformation that is applied to a system with respect to another system. Clearly, a spatial separation of the world into a region that is transformed and a region that is not leaves us with a special case of Equation (21), and can therefore not be regarded as an active version of it. Local gauge transformations can therefore not be subjected to a direct observation.

The present article thus supports the accepted view that a gauge transformation of the electromagnetic potentials together with the quantum wave-function does not imply any physical change.

The passive view of gauge transformations as changes of representation is thus seen not to conflict with our ability to construct new, successful theories using the gauge argument. The pursuit of gauge covariance is an attempt to extend our knowledge using empirical considerations together with the theoretic symmetry principle. The success of the principle is anchored in contingent properties of physical interactions. I hope that this article contributes to dispel the worries of 'the paradox of a ghost or figment of our imagination turning the wheels of the real world' (Ben-Menahem [2012]). The gauge argument should neither be understood as a miracle nor as a mathematical deduction, but as a well-calculated guess.

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Program in the History and Philosophy of Science
The Hebrew University of Jerusalem
Jerusalem, Israel
guy.hetzroni@mail.huji.ac.il

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